

Numerical Analysis | (10th Edition)

Problem

Repeat Exercise 2 using Muller's method.
Reference: Exercise 2
Find approximations to within 10⁻⁵ to all the zeros of each of the following polynomials by first finding the real zeros using Newton's method and then reducing to polynomials of lower degree to determine any complex zeros.

a. $f(x) = x^4 + 5x^3 - 9x^2 - 85x - 136$
b. $f(x) = x^4 - 2x^3 - 12x^2 + 16x - 40$
c. $f(x) = x^4 + x^3 + 3x^2 + 2x + 2$
d. $f(x) = x^5 + 11x^4 - 21x^3 - 10x^2 - 21x - 5$
e. $f(x) = 16x^4 + 88x^3 + 159x^2 + 76x - 240$
f. $f(x) = x^4 - 4x^2 - 3x + 5$
g. $f(x) = x^4 - 2x^3 - 4x^2 + 4x + 4$
h. $f(x) = x^3 - 7x^2 + 14x - 6$

Step-by-step solution

Step 1 of 85

The relevant theory to solve the given problem is provided below.
Muller's Method: It is similar to that of secant method. But, the secant method uses a line through two points on the curve to approximate the root. Muller's method uses a parabola passing through three points on the curve for the approximation.

Comment

Step 2 of 85

The derivation of Muller's method begins by considering the quadratic polynomial called the parabola of the form $P(x) = a(x - p_2)^2 + b(x - p_2) + c$ that passes through the points $(p_0, f(p_0))$, $(p_1, f(p_1))$, and $(p_2, f(p_2))$ respectively. The constants a, b , and c are obtained by applying the following conditions.
$$\begin{cases} f(p_0) = a(p_0 - p_2)^2 + b(p_0 - p_2) + c, \\ f(p_1) = a(p_1 - p_2)^2 + b(p_1 - p_2) + c, \text{ and} \\ f(p_2) = c \end{cases}$$

Comment

Step 3 of 85

By solving the equations the following situation arises
$$\begin{cases} a = \frac{(p_0 - p_2)[f(p_1) - f(p_2)] + (p_2 - p_1)[f(p_0) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_2 - p_0)}, \\ b = \frac{(p_1 - p_2)^2[f(p_0) - f(p_2)] - (p_2 - p_0)^2[f(p_1) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_2 - p_0)}, \text{ and} \\ c = f(p_2) \end{cases}$$

To determine the next approximation p_3 the quadratic formula
$$p_3 = p_2 - \left(\frac{2c}{b + \operatorname{sgn}(b)\sqrt{b^2 - 4ac}} \right)$$
 is applied

Comment

Step 4 of 85

Once the approximation p_3 is found, the procedure is reinitialized by using p_1, p_2 , and p_3 to determine the next approximation p_4 and the method is continued until a satisfactory approximation is obtained.
At each step, Muller's method involves the radical $\sqrt{b^2 - 4ac}$. So the method yields approximate complex roots when $\sqrt{b^2 - 4ac} < 0$

Comment

Step 5 of 85

a. Find the approximations to within 10^{-4} to all the zeros of the polynomial $f(x) = x^4 + 5x^3 - 9x^2 - 85x - 136$ Using Muller's method by applying Maple as shown below.
Find the first real zero:
 $\> f := x \mapsto x^4 + 5 \cdot x^3 - 9 \cdot x^2 - 85 \cdot x - 136$;
 $\> p0 := 3 : p1 := 4 : p2 := 5$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p3 := \operatorname{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$
 $p3 := 4.116630055$

Comment

Step 6 of 85

Continuation of the above
 $\> p0 := p1 : p1 := p2 : p2 := p3$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p4 := 4.123064569$
Continuation of the above
 $\> p0 := p1 : p1 := p2 : p2 := p4$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p5 := 4.123105612$

Comment

Step 7 of 85

Continuation of the above
 $\> p0 := p1 : p1 := p2 : p2 := p5$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p6 := p5 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p6 := 4.123105626$
Result: Thus, the real zero approximation is $p_6 = 4.123105626$

Comment

Step 8 of 85

Find the second real zero:
 $\> f := x \mapsto x^4 + 5 \cdot x^3 - 9 \cdot x^2 - 85 \cdot x - 136$;
 $\> p0 := -4.5 : p1 := -4.2 : p2 := -4$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p3 := \operatorname{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$
 $p3 := -4.12426968$

Comment

Step 9 of 85

Continuation of the above
 $\> p0 := p1 : p1 := p2 : p2 := p3$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p4 := -4.123109076$

Comment

Step 10 of 85

Continuation of the above
 $\> p0 := p1 : p1 := p2 : p2 := p4$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p5 := -4.123105622$
Result: Thus, the real zero approximation is $p_5 = -4.123105622$
Find the conjugate complex zeros:
 $\> f := x \mapsto x^4 + 5 \cdot x^3 - 9 \cdot x^2 - 85 \cdot x - 136$;
 $\> p0 := -3 : p1 := -2.5 : p2 := -2$;
 $\> fl := f(p0) : fl := f(p1) : f2 := f(p2)$;
 $\> c := f2$;
 $\> a := \frac{(p1 - p2) \cdot (fl - f2) - (p0 - p2) \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> b := \frac{(p0 - p2)^2 \cdot (fl - f2) - (p1 - p2)^2 \cdot (fl - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p2 - p1)}$;
 $\> p3 := \operatorname{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$
 $p3 := -3.07130 + 1.350211i$

Comment

Step 11 of 85

Continuation of the above
 $\> p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p4 := -2.36916 + 1.763701i$

Comment

Step 12 of 85

Continuation of the above
 $\> p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p5 := -2.37934 + 1.216081i$

Comment

Step 13 of 85

Continuation of the above
 $\> p6 := p5 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p6 := -2.56792 + 1.208981i$

Comment

Step 14 of 85

Continuation of the above
 $\> p7 := p6 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p7 := -2.57068 + 1.361811i$

Comment

Step 15 of 85

Continuation of the above
 $\> p8 := p7 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p8 := -2.47638 + 1.359711i$

Comment

Step 16 of 85

Continuation of the above
 $\> p9 := p8 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p9 := -2.47848 + 1.308751i$

Comment

Step 17 of 85

Continuation of the above
 $\> p10 := p9 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p10 := -2.50853 + 1.309231i$

Comment

Step 18 of 85

Continuation of the above
 $\> p11 := p10 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p11 := -2.50828 + 1.328011i$

Comment

Step 19 of 85

Continuation of the above
 $\> p12 := p11 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p12 := -2.49693 + 1.327761i$

Comment

Step 20 of 85

Continuation of the above
 $\> p13 := p12 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p13 := -2.49709 + 1.321011i$
Continuation of the above
 $\> p14 := p13 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p14 := -2.50113 + 1.321091i$

Comment

Step 21 of 85

Continuation of the above
 $\> p15 := p14 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p15 := -2.50109 + 1.323541i$

Comment

Step 22 of 85

Continuation of the above
 $\> p16 := p15 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p16 := -2.49959 + 1.323511i$

Comment

Step 23 of 85

Continuation of the above
 $\> p17 := p16 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p17 := -2.49963 + 1.322621i$

Comment

Step 24 of 85

Continuation of the above
 $\> p18 := p17 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p18 := -2.50017 + 1.322621i$

Comment

Step 25 of 85

Continuation of the above
 $\> p19 := p18 - \frac{2 \cdot c}{b + \left(\frac{b}{\operatorname{abs}(b)}\right) \cdot \operatorname{sqrt}(b^2 - 4 \cdot a \cdot c)}$
 $p19 := -2.50013 + 1.322971i$

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Continuation of the above

$$p20 := p19 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p20 := -2.49993 + 1.32294i$$

Comment

Step 27 of 85

Continuation of the above

$$p21 := p20 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p21 := -2.49995 + 1.32283i$$

Comment

Step 28 of 85

Continuation of the above

$$p22 := p21 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p22 := -2.50003 + 1.32285i$$

Comment

Step 29 of 85

Continuation of the above

$$p23 := p22 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p23 := -2.50001 + 1.32288i$$

Comment

Step 30 of 85

Continuation of the above

$$p24 := p23 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p24 := -2.49999 + 1.32286i$$

Comment

Step 31 of 85

Continuation of the above

$$p25 := p24 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p25 := -2.50000 + 1.32288i$$

Continuation of the above

$$p26 := p25 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p26 := -2.50001 + 1.32288i$$

Result: Thus, the conjugate complex zeros are given by

$$p_{25} := (-2.50000) \pm (1.32288)i$$

Comment

Step 32 of 85

b. Find the approximations to within 10^{-4} to all the zeros of the polynomial

$f(x) = x^4 - 2x^3 - 12x^2 + 16x - 40$ Using Muller's method by applying Maple as shown below.

Find the first real zero:

$$f := x \mapsto x^4 - 2 \cdot x^3 - 12 \cdot x^2 + 16 \cdot x - 40;$$
$$p0 := -4 : p1 := -3.5 : p2 := -3 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p3 := \text{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$$
$$p3 := -3.54701$$

Comment

Step 33 of 85

Continuation of the above

$$p0 := p1 : p1 := p2 : p2 := p3 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p4 := -3.54824$$

Comment

Step 34 of 85

Continuation of the above

$$p0 := p1 : p1 := p2 : p2 := p4 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p5 := -3.54823$$

Result: Thus, the real zero approximation is $p_5 = -3.54823$

Comment

Step 35 of 85

Find the second real zero:

$$f := x \mapsto x^4 - 2 \cdot x^3 - 12 \cdot x^2 + 16 \cdot x - 40;$$
$$p0 := 5 : p1 := 4.5 : p2 := 4 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p3 := \text{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$$
$$p3 := 4.38440$$

Continuation of the above

$$p0 := p1 : p1 := p2 : p2 := p3 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p4 := 4.38113$$

Comment

Step 36 of 85

Continuation of the above

$$p0 := p1 : p1 := p2 : p2 := p4 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p5 := 4.38111$$

Result: Thus, the real zero approximation is $p_5 = 4.38111$

Comment

Step 37 of 85

Find the conjugate complex zeros:

$$f := x \mapsto x^4 - 2 \cdot x^3 - 12 \cdot x^2 + 16 \cdot x - 40;$$
$$p0 := 0.6 : p1 := 0.5 : p2 := 0.4 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p3 := \text{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$$
$$p3 := 0.611194 - 1.61123i$$

Comment

Step 38 of 85

Continuation of the above

$$p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p4 := 0.449612 - 1.59427i$$

Comment

Step 39 of 85

Continuation of the above

$$p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p5 := 0.566117 - 1.35994i$$

Comment

Step 40 of 85

Continuation of the above

$$p6 := p5 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p6 := 0.722568 - 1.46500i$$

Comment

Step 41 of 85

Continuation of the above

$$p7 := p6 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p7 := 0.601192 - 1.62248i$$

Comment

Step 42 of 85

Continuation of the above

$$p8 := p7 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p8 := 0.467144 - 1.50342i$$

Continuation of the above

$$p9 := p8 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p9 := 0.568100 - 1.37601i$$

Result: Thus, the conjugate complex zeros are given by

$$p_6 \approx (0.568100) \pm (1.37601)i$$

Comment

Step 43 of 85

c. Find the approximations to within 10^{-3} to all the zeros of the polynomial

$f(x) = x^4 + x^3 + 3x^2 + 2x + 2$ Using Muller's method by applying Maple as shown below.

Find the conjugate complex zeros:

$$f := x \mapsto x^4 + x^3 + 3 \cdot x^2 + 2 \cdot x + 2;$$
$$p0 := -0.6 : p1 := -0.5 : p2 := -0.4 :$$
$$f0 := f(p0) : f1 := f(p1) : f2 := f(p2) :$$
$$c := f2 :$$
$$a := \frac{(p1 - p2) \cdot (f0 - f2) - (p0 - p2) \cdot (f1 - f2)}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$b := \frac{((p0 - p2)^2 \cdot (f1 - f2) - (p1 - p2)^2 \cdot (f0 - f2))}{(p0 - p2) \cdot (p1 - p2) \cdot (p0 - p1)} :$$
$$p3 := \text{evalf}\left(p2 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}\right)$$
$$p3 := -0.373754 - 0.738034i$$

Comment

Step 44 of 85

Continuation of the above

$$p4 := p3 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p4 := -0.327521 - 0.913486i$$

Comment

Step 45 of 85

Continuation of the above

$$p5 := p4 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p5 := -0.522649 - 1.01229i$$

Comment

Step 46 of 85

Continuation of the above

$$p6 := p5 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p6 := -0.558026 - 0.842283i$$

Comment

Step 47 of 85

Continuation of the above

$$p7 := p6 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p7 := -0.488129 - 0.835287i$$

Comment

Step 48 of 85

Continuation of the above

$$p8 := p7 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p8 := -0.480512 - 0.872929i$$

Comment

Step 49 of 85

Continuation of the above

$$p9 := p8 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p9 := -0.503922 - 0.877345i$$

Comment

Step 50 of 85

Continuation of the above

$$p10 := p9 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p10 := -0.506358 - 0.863720i$$

Repeat the above process and terminates at the following step

$$p26 := p25 - \frac{2 \cdot c}{b + \left(\frac{b}{\text{abs}(b)}\right) \cdot \text{sqrt}(b^2 - 4 \cdot a \cdot c)}$$
$$p26 := -0.500001 - 0.866028i$$

Result: Thus, the conjugate complex zeros are given by

$$p_{26} \approx (-0.500001) \pm (0.866028)i$$

Comment

Step 51 of 85

